EFFECT OF THE FINITE LENGTH OF A SLOT CHANNEL AND NONLINEARITY OF THE TEMPERATURE FIELD ON THE THERMAL EFFUSION COEFFICIENT

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As is known [1], nonisothermality of the walls of channels results in a thermomolecular pressure difference (TPD) which must be taken into account in measuring pressures in vacuum equipment. This pressure gradient was observed experimentally by Knudsen [2-4], who offered a theoretical explanation of this phenomenon for the limiting case corresponding to the free-molecular regime ($\alpha = 0$).

The other limiting case ($\alpha \to \infty$, $Kn_H \to 0$) was examined in [5, 6]. Solutions for a plane slot channel of infinite width with a unidimensional temperature distribution were obtained in [6-13]. Actual slot channels have a finite width, so it is natural to expect that the "long-range" effect described in [14, 15] and manifest at high numbers Kn_H will require appreciable corrections to the values of TPD.

In the case of a slot channel of constant width b and height H, the TPD can be determined by using Eq. (17) from [15] if we replace the displacement width b_* by the half-width of the channel, i.e., by the quantity b/2. Since $k_\nu = k_\Pi - k_\tau$, the formula for the parameter $\gamma = d\Pi_W/d\tau_W = k_\pi/k_\tau = (T_W/P_W)dP_W/dT_W$, which characterizes the thermomolecular effect, is written in the form

$$\gamma = \frac{Q_{\nu\infty}(\alpha) - Q_{\tau\infty}(\alpha) + 2\varepsilon_0[\Delta_\nu(\alpha) - \Delta_\tau(\alpha)]}{Q_{\nu\infty}(\alpha) + 2\varepsilon_0\Delta_\nu(\alpha)}, \tag{1}$$

where $\Pi_W = \nu_W + \tau_W$; $\varepsilon_0 = H/b$.

Table 1 shows the results of calculations with this formula for a plane channel of infinite width ($\varepsilon_0 = 0$). The results were obtained using the values $Q_\nu^{(0)}(\alpha) = Q_{\nu\infty}(\alpha)$, $Q_\tau^{(0)}(\alpha) = Q_{\tau\infty}(\alpha)$ tabulated in [11, 14] and are close to the data in [10], which was calculated by the method of iteration of moment solutions for the ES-model. At small α, the values of γ found here are somewhat higher than those found in [10]. It must be noted that their insertion into the formula

$$\Lambda(\alpha) = \frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{\gamma(\alpha)}, \quad \left(\frac{dP}{dT} = \gamma\frac{P}{T}\right), \tag{2}$$

which evaluates the ratio of the pressures at the ends of a plane channel from the given temperature ratio, results in curves $\Lambda(\alpha)$ lying somewhat higher in the region of small α than the curves obtained in [11]. This difference is probably connected with the finite width b of the test channel.

To evaluate the effect of a finite channel width on the thermal effusion coefficient γ, we will use Eq. (1). As was shown in [15], at $\alpha \to 0$, $\Delta_\tau \approx \Delta_\nu \approx (4\pi\alpha)^{-1}$. Thus, for large Knudsen numbers Kn_H, when the "long-range" effect is particularly evident, the asymptotic formula for γ has the form

$$\gamma = \frac{Q_{\nu\infty}(\alpha) - Q_{\tau\infty}(\alpha)}{Q_{\nu\infty}(\alpha) + \varepsilon/2\pi\alpha}. \tag{3}$$

At $\alpha \lesssim 0.01$ in this formula, the asymptotic representations of the functions $Q_{\nu,\tau\infty}(\alpha)$ can be taken from [15]. This introduces a relative error no greater than 1%. Then

$$\gamma = \frac{\ln\alpha + 0{,}5}{2(\ln\alpha - 0{,}5) + \varepsilon_0/2\alpha\sqrt{\pi}}. \tag{4}$$

At $0.01 < \alpha < 0.1$, to attain a high degree of accuracy, calculations should be performed with Eq. (3) using tabulated values of the functions $Q_{\nu,\tau\infty}(\alpha) = Q_{\nu,\tau}^{(0)}(\alpha)$ [16]. In the case of low values of theparameter ε_0, such an approach would obviously be consistent with the above

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TABLE 1. Comparative Values of the Thermal Effusion Coefficients γ in Relation to the Parameter α (at $\alpha \to 0$, $\gamma \to 0.5$)

Results of calculation	α	0,01	0,1	0,2	0,3	0,5	0,7	1,1	1,5	2	3	5	7	10
	γ	0,4051	0,3405	0,3057	0,2803	0,2424	0,2141	0,1729	0,1437	0,1170	0,0825	0,0474	0,0305	0,0184
From [11]	γ	0,4026	0,3356	0,3008	0,2729	0,2428						0,0473		

asymptotic evaluation of the parameters Δ_ν and Δ_τ. At moderately low values of the parameters α and ε_0, the formulas obtained in [14, 15] for $\Delta_{\nu,\tau}(\alpha)$ can be recommended for determining the coefficient γ. The values of the integrals $I_m(x)$ tabulated in [16] can be used in these calculations.

The results of calculations with Eqs. (3) (for $\alpha \geq 0.01$) and (4) (for $\alpha < 0.01$) shown in Fig. 1 illustrate the effect of the finite width of the channel on the thermal effusion coefficient $\gamma(\alpha, \varepsilon_0)$ which characterizes the TPD. This effect is sharply intensified with a reduction in α. It was suggested in [9] that the TPD is either independent or only slightly dependent on channel geometry, although a calculation of the thermal effusion coefficient for a circular capillary tube [13] showed that the corresponding curve $\gamma(\alpha)$ lies significantly to the right of the thermal effusion curve for the slot channel. This result can be explained by the fact that the momentum flow from the walls due to the temperature gradient is proportional to the area of the walls, while the momentum flow from the pressure gradient is proportional to the area of the cross section. The ratio of these areas for identical cross-sectional dimensions (with equal values of α) is four times greater for a circular capillary tube of diameter $d = H$ than for a plane channel of infinite width ($b = \infty$, $\varepsilon_0 = 0$). Thus in a circular capillary tube, a regime close to the free-molecular regime is maintained at significantly higher values of α. A feature of the plane channel of finite width relevant to this discussion is that different sections of the walls of this channel play different roles during thermal effusion. As was shown in [14, 15], the effect of the lateral walls of the channel on gas flow in the main zone is appreciable only at sufficiently large numbers $Kn_b = \lambda/b$, when the "long-range" effect is manifest. It is evident from Fig. 1 that this effect becomes significant at high values of ε_0 ($Kn_b = \varepsilon_0 Kn_H$) with an increase in α.

Until now, the temperature field of the channel walls has been assumed to be unidimensional and linear when we determined the thermal effusion coefficient γ. It is this very property that makes it possible to establish an unambiguous relationship between the temperature and pressure gradients with a given geometry of the channel cross section. However, the macroscopic gas velocity $\langle \bar{u} \rangle$ is not unambiguously determined by the gradients $\nabla \nu_w$ and $\nabla \tau_w$. It also depends on the higher derivatives of the functions, which is one more manifestation of the "long-range" effect so characteristic of low-density gas flows in slot channels. Thus, as the second problem we examine the effect of the nonsolenoidal character of the vector $\nabla \tau_w$ on the value of the thermal effusion coefficient γ. To do this, we equate the expansion of the macroscopic velocity $\langle \bar{u} \rangle$ to zero [14], i.e., we write the equation

$$Q_\nu^{(0)}(\alpha)\nabla\nu_w + Q_\tau^{(0)}(\alpha)\nabla\tau_w + \mathrm{Kn}_L^2\,[Q_\nu^{(2)}(\alpha)\nabla\Delta\nu_w + Q_\tau^{(0)}(\alpha)\nabla\Delta\tau_w] + O(\mathrm{Kn}_L^4) = 0 \quad (\Delta = \nabla^2), \tag{5}$$

from which we find the function $\nu_w(\xi_1, \xi_2)$. At low numbers Kn_L, Eq. (5) will be singularly perturbed. However, in the absence of "boundary" layers in the part of the slot channel being studied, the solution of the equation can be constructed in the form of a regular expansion in powers of the small parameter Kn_L^2. Following the terminology in [17], this expansion is an asymptotic expansion, i.e.,

$$\nu_w = \nu_w^{(0)} + \mathrm{Kn}_L^2\,\nu_w^{(2)} + O(\mathrm{Kn}_L^4),$$
$$\nu_w^{(0)} = -\frac{Q_\tau^{(0)}(\alpha)}{Q_\nu^{(0)}(\alpha)}\,\tau_w, \tag{6}$$
$$\nu_w^{(2)} = \frac{1}{Q_\nu^{(0)}(\alpha)}\left[\frac{Q_\tau^{(0)}(\alpha)}{Q_\nu^{(0)}(\alpha)}\,Q_\nu^{(2)}(\alpha) - Q_\tau^{(2)}(\alpha)\right]\Delta\tau_w.$$

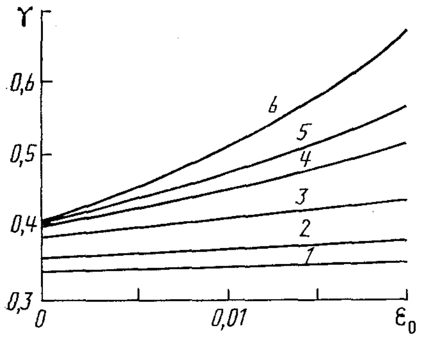

Fig. 1

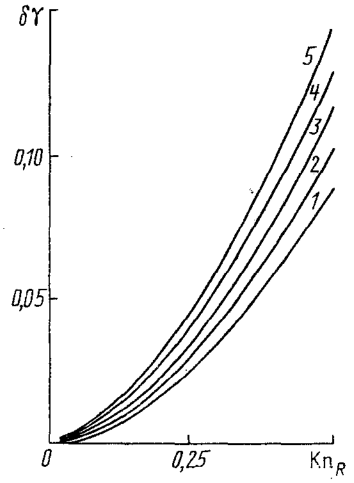

Fig. 2

Fig. 1. Effect of the finite width of a channel on the thermal effusion coefficient $\gamma(\alpha, \varepsilon_0)$: 1) $\alpha = 0.01$; 2) 0.1; 3) 0.2; 4) 0.3; 5) 0.5; 6) 0.7.

Fig. 2. Dependence of the change in the thermal effusion coefficient $\delta\gamma$ on the number Kn_R constructed from the radius of curvature of isotherms passing through the given point of the wall and the relative height of the slot channel H/R: 1) $H/R = 5 \cdot 10^{-3}$; 2) 10^{-2}; 3) $2 \cdot 10^{-2}$; 4) $3 \cdot 10^{-2}$; 5) $4 \cdot 10^{-2}$.

The thermal effusion coefficient characterizing the thermomolecular effect is

$$\gamma = \frac{\nabla \Pi_w}{\nabla \tau_w} = 1 + \frac{\nabla v_w}{\nabla \tau_w} = \gamma_0 + \mathrm{Kn}_L^2 \gamma_2 + O(\mathrm{Kn}_L^4). \tag{7}$$

The correction γ_2, proportional to $\nabla \Delta \tau_w$, shows in particular that thermal effusion is dependent on the curvature of the isotherms on the walls of the slot channel. In the case of a unidimensional temperature field, the correction reflects the effect of the nonlinearity of the function $\tau_W(\xi_1)$. In fact, if the temperature field in a sufficiently large neighborhood of point A is described by the function $\tau_w = \vartheta R/R_0$, where R is the polar coordinate of the test point in the plane of the plate, then

$$\Delta \tau_w = \left(\frac{d^2}{dR_1^2} + \frac{1}{R_1}\frac{d}{dR_1}\right)\tau_w = \nu \frac{R_0}{R} = \tau_w \left(\frac{R_0}{R}\right)^2 \quad \left(R_1 = \frac{R}{R_0}\right). \tag{8}$$

As the linear scale of L here, we chose the characteristic radius R_0. Insertion of (8) into (7) gives $[\nabla \Delta \tau_w = -\vartheta (R_0/R)^2]$

$$\gamma = \gamma_0 + \delta\gamma, \ \delta\gamma = \mathrm{Kn}_R^2 \frac{Q_\tau^{(2)}(\alpha) - [1 - \gamma_0(\alpha)]\, Q_v^{(2)}(\alpha)}{Q_v^{(0)}(\alpha)}, \ \mathrm{Kn}_R = \frac{\lambda}{R}. \tag{9}$$

The coefficients in the last formula are functions of the parameter

$$\alpha = \frac{\sqrt{\pi}}{3}\frac{H}{\lambda} = \frac{\sqrt{\pi}}{3}\frac{H}{R}\frac{1}{\mathrm{Kn}_R}.$$

Thus, $\delta\gamma(\mathrm{Kn}_R, H/R)$, i.e., the corresponding correction, depends on the Knudsen number constructed from the radius of curvature of the isotherm passing through the given wall point, and it also depends on the relative height of the slot channel. At $\alpha \to 0$, Eq. (9) can be simplified by replacing the coefficients by their asymptotic representations. Then

$$\delta\gamma = \frac{\pi}{9}\left(\frac{H}{R}\right)^2 \left[\frac{1}{2} + \gamma(\alpha)\right] \frac{Q_v^{(2)}(\alpha)}{\alpha^2 Q_v^{(0)}(\alpha)}. \tag{10}$$

The dependence of $\delta\gamma$ on the number Kn_R and H/R calculated from this formula is shown in Fig. 2.

We can similarly calculate the correction $\delta\gamma$ due to the nonlinearity of the unidimensional temperature field $\tau_W = \tau_W(\xi)$. In this case $\nabla \Delta \tau_w = d^3\tau_w/d\xi^3$. Thus, as before, Eqs. (9) and

(10) can be used if the number Kn_R and the radius of curvature of the isotherm R in these equations are replaced by the number Kn_l and the linear scale l, respectively:

$$\mathrm{Kn}_l = \frac{\lambda}{l}, \quad l = \sqrt{\left|\frac{dT/dx}{d^3T/dx^3}\right|}. \tag{11}$$

The results shown in Fig. 2 and the results calculated with Eqs. (9) and (10) should be multiplied by ±1 at $(d^3T/dx^3)/(dT/dx) \gtreqless 0$. It is somewhat surprising that the sign of the correction δ_γ is opposite the sign of the ratio ($\nabla\Pi_w=0$). This seeming paradox is explained by the fact that $|Q_\tau^{(2)}(\alpha)| > |Q_\nu^{(2)}(\alpha)|$. Thus with a constant pressure the flow of a low-density gas in the cross section of the channel corresponding to a zero local temperature gradient ($\nabla\tau_w=0$) is directed opposite the vector $\nabla\Delta\tau_w$. In the case $\nabla\Pi_w=0$ and $\nabla\Delta\tau_w=0$ the directions of the gas flow and the temperature gradient coincide.

In fact

$$\int_{-0,5}^{0,5} \langle \bar{u} \rangle \, d\zeta = Q_\nu^{(0)} \nabla\Pi_w + Q_T^{(0)} \nabla\tau_w + \mathrm{Kn}_L^2 [Q_\nu^{(2)} \nabla\Delta\Pi_w + Q_T^{(2)} \nabla\Delta\tau_w], \tag{12}$$

$$Q_T^{(0)}(\alpha) = Q_\tau^{(0)} - Q_\nu^{(0)} > 0, \quad Q_T^{(2)}(\alpha) = Q_\tau^{(2)} - Q_\nu^{(2)} < 0.$$

NOTATION

H, height of slot channel; L, linear scale in the middle plane; $Kn_H = \lambda/H$, $Kn_L = \lambda/L$; λ, mean free path of the molecules; $\alpha = \sqrt{\pi H/3\lambda}$; x_1, x_2, x_3, Cartesian coordinate system; x_1, distance along the contour; x_2, distance from the contour of the channel; x_3, distance from the middle plane of the slot channel; $\xi = x_1/H$; $\eta = x_2/H$; $\zeta = x_3/H$; n_*, characteristic value of the numerical density of the molecules; n, numerical value of the molecules; $\nu = (n - n_*)/n_*$; $\tau = (T - T_*)/T_*$; T_*, characteristic temperature; T, temperature; $k_\nu = \partial\nu/\partial\xi$; $k_\tau = \partial\tau/\partial\xi$; $\langle u \rangle$, macroscopic velocity of gas directed along the axis ξ; b_*, boundary layer displacement width; $\varepsilon_0 = H/b$; $k_\Pi = \frac{1}{P}\frac{\partial P}{\partial \xi} = k_\nu + k_\tau$; P, pressure; $\gamma = \frac{T}{P}\frac{dP}{dT}$ — thermal effusion coefficient; $Q_{\nu\tau}(\alpha)$, $\delta Q_{\nu\tau}(\alpha)$, $\Delta_\nu(\alpha)$, and $\Delta_\tau(\alpha)$, coefficients determined by Eqs. (8), (9), and (17) from [15]; $I_m(x) = \int_0^\infty v^n \exp(-v^2 - x/v)\, \partial v$; $\Lambda = P_1/P_2$, ratio of pressures on the ends of the plane channel; $\Pi = \nu + \tau$. Indices: w, on the wall.

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